# Shear and Moment Response of the Airplane Wing to Nonstationary Turbulence

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## Theme

THIS paper presents the analysis of shear and moment of the airplane wing due to nonstationary atmospheric turbulence by means of evolutionary cross spectrum approach. The airplane is considered to be a multimodal linear system consisting of rigid and flexible motions. The nonstationary turbulence is assumed to be the product of a deterministic envelope function and a stationary random process. Shear and moment responses at arbitrary wing sections can be obtained in terms of time-varying spectral density function (evolutionary spectrum) and mean square values. Our analysis includes the conventional stationary solutions as the special case.

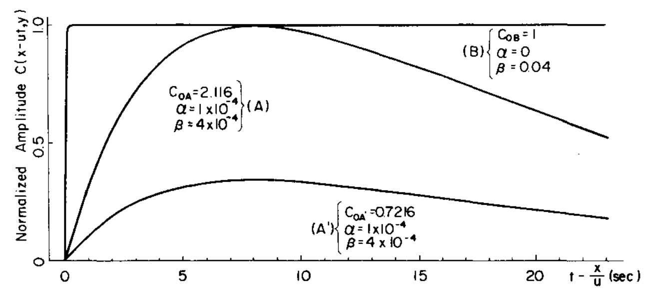


Fig. 2 Profile of the envelope function.

## Contents

Necessity to analyze the nonstationary structural response has grown out of our experiences that various phenomena such as the airplane response in the gust, ground buildings' response under the earthquakes, overshoot of vibration amplitude in reciprocating or rotary machinery, and so forth, show mostly the nonstationary time-wise trend. Within the scope of aeronautical problems, most flight records of acceleration in the turbulence are said to be of nonstationary random nature, and in the worst case, a big airplane suffers structural catastrophe just after entering the turbulent field behind the high mountain in the clear sky.

The data piled up for years by observations or experiments are indicative of importance of nonstationary analysis from the viewpoint of not only fatigue damage but also ultimate strength of the structure under dynamic loading. But the analysis of many mechanical systems' transient response in terms of spectral concept was seldom done because of difficulties in various aspects such as engineering feasibilities, mathematical representations, and so forth. In the previous papers[1,2] the author has derived the evolutionary spectrum of multi-degree-of-freedom systems' response and shown its usefulness to describe the multimodal airplane motions.

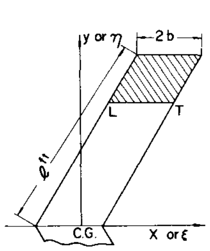


Fig. 1 Coordinates of the swept-back wing configuration.

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In this paper, the author intends to work out the shear and moment response of the airplane wing which are directly related to stress or strain of the wing structure and will yield the valuable information in the incipient design phase of the airplane.

The numerical examples are carried out for the airplane configuration pictured in Fig. 1. The shear and moment of the section L-T can be obtained by integration of acting forces on the wing within the hatched area of Fig. 1. The deterministic envelope functions used in the calculation are shown as curves A and B of Fig. 2. The response for $t \to \infty$ in case B gives the stationary solution. The evolutionary spectrum of the moment at the wing root is pictured in Fig. 3 which shows the time-varying spectral density function. While $t$ is small, the spectrum curves have flat portion below $\omega = 1$ rad/sec and sharp decline in the level for upper frequency range. But as time $t$ gets large, they turn to have double peaks, one at about $\omega = 0.1 \sim 1$ rad/sec and the other at $\omega = 13.6$ rad/sec, the former exhibits the effect of rigid body motions and Dryden's spectrum, the latter the first

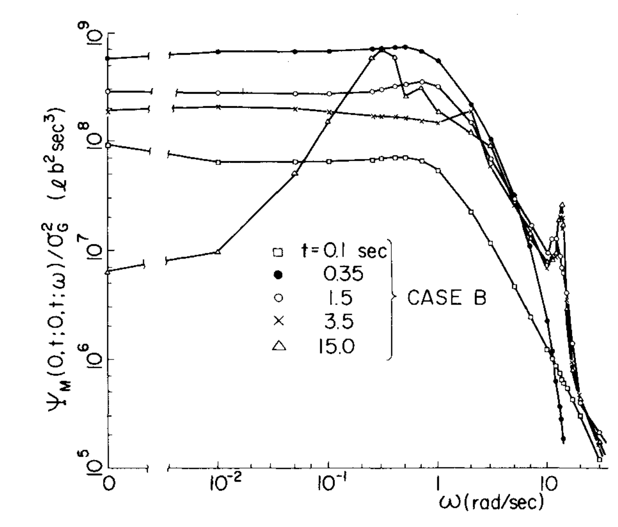


Fig. 3 Evolutionary spectrum of moment at the wing root for case B.

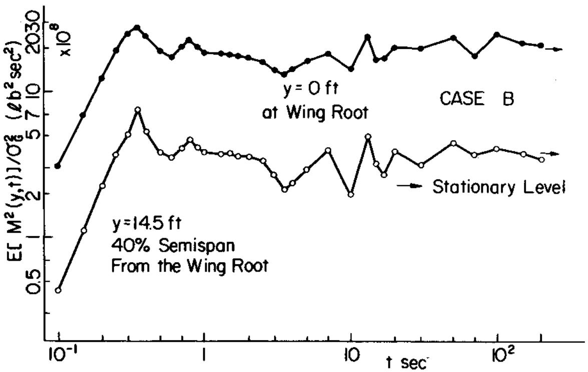


**Fig. 4 Mean square moment for case B.**

bending vibration. The evolutionary spectrums of shear forces look similar to those of moment. Over-all response levels are given in terms of mean square moment that can be obtained by the numerical integration of evolutionary spectrum curves. The result at the wing root, $y = 0$ ft, is graphed in Fig. 4, where mean square values reach a transient maximum and with some fluctuations go to the stationary level. From Figs. 3 and 4 we see that the rigid modes play a predominant role in the evolutionary spectrum at the time of maximum mean-square moment. On the other hand in the stationary moment response, we see the effect of flexibility in addition to rigid movements. Therefore, the search of the stationary solutions only is not adequate in the sense that it overlooks the transient maximum when the airplane enters the step-like gust field.

*Appendix*

It is convenient that the normalizing constant $C_O$ of the amplitude profile should be chosen so that the maximum of the curve takes the unit level. In this report, curve A of Fig. 2 is used, but in the previous papers[1,2] the curve of A′ was taken because of an unduly placed parenthesis in the computer program.

In order to compare the spectrum and mean square values of case A with those of case B, it is necessary that the corrective factor $(C_{OA}/C_{OA'})^2 = 8.60$ should be multiplied to the vertical scale of Figs. 3–8 of Ref. 2 and Figs. 2 and 3 of Ref. 1. Also $C_{OA}/C_{OA'} = 2.93$ to Fig. 4 of Ref. 1. As long as we take the same $\alpha$ and $\beta$ values, the evolutionary spectrum has the identical frequency distribution, but its level depends on the normalizing constant $C_O$, so the spectrum of case A can be obtained from the relation

$$\Psi_A = \Psi_{A'}(C_{OA}/C_{OA'})^2$$

It is recommended that we use the unit maximum for case A so that we may compare the response level of case A with that of case B.

## References

[1] Fujimori, Y. and Lin, Y. K., "Analysis of Airplane Response to Nonstationary Turbulence Including Wing Bending Flexibility II," *AIAA Journal*, Vol. 11, No. 9, Sept. 1973, pp. 1343–1345.

[2] Fujimori, Y. and Lin, Y. K., "Analysis of Airplane Response to Nonstationary Turbulence Including Wing Bending Flexibility," *AIAA Journal*, Vol. 11, No. 3, March 1973, pp. 334–339